Application of Holography to Panel Flutter

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An analytical approach which outlines the use of holographic interferometry to measure the deflection shape of a fluttering panel is presented. The approach relies on a differential holographic technique which has been demonstrated experimentally on other structures. For single-frequency flutter, two differencial holograms are all that are required to determine completely the amplitudes and phases of all modes participating in the flutter. For nonlinear or nonsteady flutter, it is possible to make a single differential hologram which yields the instantaneous velocity of the flutter surface at a given time. A single hologram of this type will provide the experimental data to compare with theory regarding the spatial distribution of the flutter mode.

Introduction

HOLOGRAPHIC interferometry[1] has emerged as a powerful experimental tool in applied mechanics. Since its early beginnings, holographic interferometry has been applied to vibrating objects.[2] Early applications to vibrations were time-average holography[2,3] and stored-beam (real-time) holography.[4] Other related applications have included transient response measurements[5] and transverse wave propagation studies.[6,7]

A general restriction of the previous work is that for the most part one is limited to the study of small-amplitude motions, i.e., amplitudes on the order of a few wavelengths of light. Large-amplitude motions cause the interference fringes to crowd together, which results in problems of resolution. Recently, it was proposed and demonstrated[8] that the limitation of small-amplitude motions could be overcome by a differential pulsing technique. By making two exposures in rapid succession, only a few interference fringes are formed. This method of differential pulsing eliminates the need to restrict holographic studies to small-amplitude motions. Thus, it becomes possible to explore the application of holography to panel flutter, where the amplitudes of motion are relatively large. Accordingly, it is the purpose of this paper to demonstrate that differential pulsed holography can be used to measure deflections during panel flutter.

The analysis shows that for flutter at a single frequency, two differential holograms made at separate times are sufficient to determine the amplitude and phase of all of the modes participating in the flutter. For all types of flutter, a single differential hologram will provide the experimental data for comparison with analysis, if the structural analyst is willing to compute panel transverse velocity (as a function of space) instead of panel deflection. Since the technique uses a pulsed ruby laser operating in a double-pulse mode, it is unaffected by ambient vibration and noise, or aerodynamic fluctuations in the flow.† Holography applied to panel flutter possesses the unique advantage that it provides experimental data as a function of space (over the entire panel) instead of the more customary point measurements. The technique outlined herein appears very promising, and now that the analytical background has been established it will be only a matter of time before holography is used to measure panel flutter in actual practice.

Received June 25, 1976; revision received Sept. 20, 1976.

Index categories: Aeroelasticity and Hydroelasticity; Structural Dynamic Analysis.

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†See, for example, Ref. 9, in which pulsed differential holograms of a vibrating plate at 2000°F were made through thermal gradients and a turbulent boundary layer.

Infinite Strip Plate

Defining the Problem

Holography appears potentially useful for virtually all types of panel flutter, regardless of panel shape and/or boundary conditions. However, the essential ideas of the technique are demonstrated most readily by a simple example. Thus, consider flow past an infinite strip plate (Fig. 1) which is simply-supported on its boundaries.

When flutter occurs, the transverse deflection of the plate can be written as an eigenfunction expansion[10]

$$w(x,t) = \sum_{m=1}^{\infty} q_m(t)\phi_m(x) \tag{1}$$

where

$$q_m(t) = C_m \cos(\omega t + \theta_m) \tag{2}$$

or, equivalently,

$$q_m(t) = A_m \cos\omega t + B_m \sin\omega t \tag{3}$$

and

$$\phi_m(x) = \sin m\pi x/L \tag{4}$$

where ϕ_m are the undamped normal modes and ω is the flutter frequency.

The question posed for holography is as follows: can holographic interferometry be used to measure the deflection $w(x,t)$, and in particular can it be used to determine the amplitude coefficients A_m and B_m? The answer is affirmative, as discussed in the next few paragraphs.

Differential Pulsing Technique

In making the hologram, use will be made of the differential technique described previously.[8] The setup of the

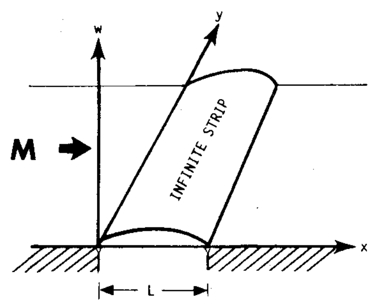

Fig. 1 Flow past an infinite strip plate.

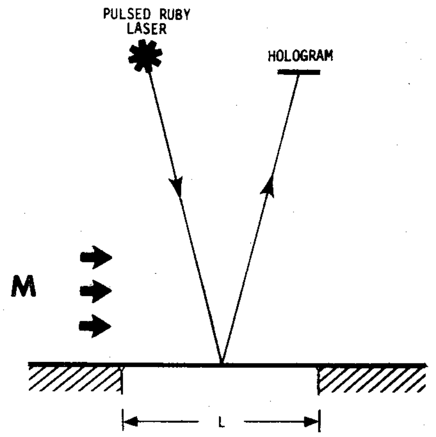

Fig. 2 Schematic arrangement of laser and panel.

laser is shown in Fig. 2. The first exposure of the hologram is made by pulsing the laser once, at time t_1, and the second exposure of the hologram is made by a second light pulse, at a time $t_1+\Delta$. When the hologram is developed and reilluminated, the interference fringe pattern which forms is a measure of the relative displacement of the fluttering panel between exposures. Thus, a photograph of this double-exposure hologram will give a fringe pattern representing the difference in displacement

$$w_2-w_1=w(x,t_1+\Delta)-w(x,t_1) \tag{5}$$

Using Eq. (1) and (3) in Eq. (5) gives

$$w_2-w_1=\sum_{m=1}^{\infty}\Big\{A_m\Big[\cos\omega(t_1+\Delta)-\cos\omega t_1\Big]$$

$$+B_m\Big[\sin\omega(t_1+\Delta)-\sin\omega t_1\Big]\Big\}\phi_m(x) \tag{6}$$

We select the time delay Δ (between exposures) such that

$$\omega\Delta<<1 \tag{7}$$

Equation (7) means that the time delay Δ must be substantially less than the period of the flutter motion. Then Eq. (6) can be expanded to give

$$w_2-w_1=\omega\Delta\sum_{m=1}^{\infty}[-A_m\sin\omega t_1+B_m\cos\omega t_1]\phi_m(x)+0(\omega\Delta)^2 \tag{8}$$

In Eq. (8). the times t_1 and Δ are known, the left-hand side (w_2-w_1) has been recorded holographically, and it is assumed that the flutter frequency ω has been measured with a conventional transducer of some sort. (The time t_1 can be obtained from the signal of such a transducer.)

Determination of the Amplitude Coefficients

Consider the hologram and w_2-w_1 for a moment. From a photograph of the interference pattern, one can count the fringes and draw a curve through the data, giving w_2-w_1 as a function of x. (Although the fringe numbers are not monotonic, for most vibration problems the fringe data are reasonably smooth and are interpreted readily. See Ref. 6 through 9, for example). Let this curve of data points be represented by

$$w_2-w_1=N^{(1)}(x) \tag{9}$$

where the superscript (1) indicates an exposure at time t_1.

The experimental curve for w_2-w_1 can be expanded readily in terms of the eigenfunctions, $\phi_m(x)$. Thus, we have (from the experimental data)

$$w_2-w_1=N^{(1)}(x)=\sum_{m=1}^{\infty}N_m^{(1)}\phi_m(x) \tag{10}$$

or in our particular example

$$N^{(1)}(x)=\sum_{m=1}^{\infty}N_m^{(1)}\sin\frac{m\pi x}{L} \tag{11}$$

Upon comparing Eq. (8) and (10) and taking account of the fact that the eigenfunctions are linearly independent, we have the relation

$$N_m^{(1)}=\omega\Delta[-A_m\sin\omega t_1+B_m\cos\omega t_1] \tag{12}$$

In Eq. (12) all of the factors are known to the experimenter except the coefficients A_m and B_m which we are trying to determine. That is, Eq. (12) is one equation with two unknowns, A_m and B_m.

To obtain another relation between A_m and B_m, suppose that we make another differential pulse hologram at times t_2 and $t_2+\Delta$. Following the same steps as before, we have

$$N^{(2)}(x)=\sum_{m=1}^{\infty}N_m^{(2)}\phi_m(x) \tag{13}$$

and

$$N_m^{(2)}=\omega\Delta[-A_m\sin\omega t_2+B_m\cos\omega t_2] \tag{14}$$

Eq. (12) and (14) now can be solved simultaneously to determine the unknowns A_m and B_m.

Performing this operation yields

$$A_m=\frac{1}{\omega\Delta D}\{-N_m^{(1)}\sin\omega t_1\cos\omega t_2+N_m^{(2)}\sin\omega t_2\cos\omega t_1\} \tag{15a}$$

$$B_m=\frac{1}{\omega\Delta D}\{-N_m^{(2)}\sin\omega t_1\cos\omega t_2+N_m^{(1)}\sin\omega t_2\cos\omega t_1\} \tag{15b}$$

where D is the determinant

$$D=\cos\omega t_1\sin\omega t_2-\sin\omega t_1\cos\omega t_2$$

$$D=\sin\omega(t_2-t_1) \tag{16}$$

Note that the determinant vanishes when $\omega(t_2-t_1)=k\pi$ (k integer) which means that the two times are 180° apart in phase and thus give the same information on both holograms.

The vanishing of the determinant easily can be avoided by proper choice of the time t_1 and t_2. A time base can be provided by the signal from a vibration pickup which monitors the panel displacement.

Once the coefficients A_m and B_m have been found from Eq. (15) the problem is solved completely since we can write

$$w(x,t)=\sum_{m=1}^{\infty}[A_m\cos\omega t+B_m\sin\omega t]\phi_m(x) \tag{17}$$

from Eq. (1) and (3) to find the deflection at any time t or location x.

Finding the Expansion Coefficients N_m

The coefficients $N_m^{(1)}$ and $N_m^{(2)}$ are determined simply by expanding the experimental curves $N^{(1)}(x)$ and $N^{(2)}(x)$ in terms of the eigenfunctions $\phi_m(x)$. Since the eigenfunctions are orthogonal, we have

$$\int_0^L\phi_i(x)\phi_j(x)\mathrm{d}x=\delta_{ij} \tag{18}$$

where δ_{ij} is the Kronecker delta. Equation (18) can be used with Eq. (10) and (13) to give

$$N_m^{(1)} = \int_o^L N^{(1)}(x)\phi_m \mathrm{d}x \tag{19a}$$

and

$$N_m{}^{(2)} = \int_o^L N^{(2)}(x)\phi_m(x)\mathrm{d}x \tag{19b}$$

respectively.

The integrals (19) can be evaluated readily, for example by a simple computer program. Recall that the functions $N^{(1)}(x)$ and $N^{(2)}(x)$ are experimental curves which can be fitted with a polynomial or otherwise put into a readily integrable form.

Other Panels, Nonlinear, or Nonsteady Flutter

Extension to Other Panels and Boundary Conditions

The ideas just presented can be extended readily to other panel geometries and boundary condtitions. For example, in the case of a two-dimensional panel we have

$$w(x,y,t) = \sum_m \sum_n q_{mn}(t)\phi_{mn}(x,y) \tag{20}$$

where $\phi_{mn}(x,y)$ are the two-dimensional eigenfunctions and $q_{mn}(t)$ is given by

$$q_{mn}(t) = A_{mn}\cos\omega t + B_{mn}\sin\omega t \tag{21}$$

As in the previous case, we have

$$A_{mn} = \frac{1}{\omega\Delta D}\{-N_{mn}{}^{(1)}\sin\omega t_1 \cos\omega t_2 + N_{mn}{}^{(2)}\sin\omega t_2\cos\omega t_1\} \tag{22a}$$

and

$$B_{mn} = \frac{1}{\omega\Delta D}\{-N_{mn}{}^{(2)}\sin\omega t_1 \cos\omega t_2 + N_{mn}{}^{(1)}\sin\omega t_2\cos\omega t_1\} \tag{22b}$$

where ω, Δ, and D were defined previously and

$$N_{mn}{}^{(1)} = \int_o^b \int_o^a N^{(1)}(x,y)\phi_{mn}(x,y)\mathrm{d}x\mathrm{d}y \tag{23a}$$

$$N_{mn}{}^{(2)} = \int_o^b \int_o^a N^{(2)}(x,y)\phi_{mn}(x,y)\mathrm{d}x\mathrm{d}y \tag{23b}$$

a and b being the panel dimensions.

The functions $N^{(1)}(x,y)$ and $N^{(2)}(x,y)$ are determined experimentally from the holograms, which are recorded differentially at times t_1 and t_2, respectively.

In many practical examples, the panel structure is such that the eigenfunctions are separable, i.e.,

$$\phi_{mn}(x,y) = F(y)\phi_m(x) \tag{24}$$

where $F(y)$ is just a function of the spanwise coordinate y and the modal decomposition is in the streamwise direction x. For example when a rectangular panel is simply-supported on all four boundaries we have (for flow in the x direction)

$$\phi_{mn}(x,y) = \sin(m\pi x/a)\sin(\pi y/b) \tag{25}$$

where only one mode participates in the spanwise direction y, and the $\phi_m(x)$ are just $\sin(m\pi x/a)$.

Extension to Nonlinear Panel Flutter

Thus far the analysis assumes that the functions $q_m(t)$ are *harmonic* in time, with a flutter frequency ω. This single frequency approximation is always assumed in a linear analysis of the flutter problem. In many flutter experiments, the nonlinear limit cycle which does occur is only weakly nonlinear, which makes the single-frequency analysis adequate. A more general case occurs when the functions $q_m(t)$ are just *periodic* in time, and thus contain many harmonics.

For nonlinear flutter of our strip plate (Fig. 1) we have

$$w(x,t) = \sum q_m(t)\phi_m(x)$$

where $q_m(t)$ is periodic in time,

i.e., $$q_m(t) = q_m(t+T)$$

where T is the *period* of the motion.

Now a periodic function $q_m(t)$ can be expanded in a Fourier series in time, giving

$$q_m(t) = \sum_{k=o}^{K}\left[A_{mk}\cos k\omega t + B_{mk}\sin k\omega t\right] \tag{26}$$

where $\omega = (2\pi/T)$ is the fundamental frequency. By making several holograms at various times, the individual harmonic coefficients A_{mk}, B_{mk} can be determined. For example, from the first hologram, at time t_1 we have

$$N_m^{(1)} = \omega\Delta\sum_{k=o}^{K}[-kA_{mk}\sin k\omega t_1 + kB_{mk}\cos k\omega t_1] \tag{27a}$$

A second hologram, made differentially at time t_2 gives

$$N_m^{(2)} = \omega\Delta\sum_{k=0}^{K}[-kA_{mk}\sin k\omega t_2 + kB_{mk}\cos k\omega t_2] \tag{27b}$$

Similarily, from the i^{th} hologram at time t_i we have

$$N_m{}^{(i)} = \omega\Delta\sum_{k=o}^{K}[-kA_{mk}\sin k\omega t_i + kB_{mk}\cos k\omega t_i] \tag{27c}$$

If $2K$ differential holograms are made, a set of equations of the form (27) can be solved by matrix manipulations to give the $2K$ unknowns A_{mk} and B_{mk}. Although this process may seem very laborious, in practice one very well could truncate the series with only one or two harmonics beyond the fundamental, since nonlinear flutter often involves but a few harmonics.

Unsteady Flutter: Velocity Measurements

Up to this point we have used holography only as a tool for providing the relative deflection $w_2 - w_1$ between exposures of the hologram. Most of the preceding analysis has been aimed at translating the holographic information into terms understandable for the structural dynamicist. Now let us take a different approach and see how the structural aspects can be altered to suit the holography.

Consider a general panel deflection, represented by $w(x,y,t)$ where x and y are the space variables and t is time. Now consider a differential pulse hologram, made at times t_1 and $t_1+\Delta$. The reilluminated hologram provides interference fringes that are a measure of relative displacement

$$w_2 - w_1 = w(x,y,t_1+\Delta) - w(x,y,t_1) \tag{28}$$

For small time delays Δ we can expand the right-hand side of Eq. (28) in a Taylor series, giving

$$w_2 - w_1 = w(x,y,t_1) + \frac{\partial w}{\partial t}\Big|_{t_1} \Delta + \tfrac{1}{2}\frac{\partial^2 w}{\partial t^2}\Big|_{t_1} \Delta^2 + O(\Delta^3) - w(x,y,t_1) \qquad (29)$$

or

$$w_2 - w_1 = \dot{w}(x,y,t_1)\Delta + O(\Delta^2) \qquad (30)$$

(Equation (30) is the generalization of Eq. (8) discussed previously.)

At all locations where the transverse velocity $\dot{w}$ of the panel is nonzero, we can choose Δ small enough so that terms of $O(\Delta^2)$ are negligible. Then our differential hologram, which records $w_2 - w_1$, can be used to measure velocity, since the fringe pattern will be directly proportional to $\dot{w}$, at time t_1.

Now all we need to do is to ask the structural analyst to calculate the velocity $\dot{w}(x,y,t)$ instead of the more familiar deflection w. By recording the velocity at a time t_1 (as a function of x and y) we will provide the structures engineer with a well-defined quantity which he can compare with his analysis.

A single hologram, made by differential pulsing, will tell the structural analyst whether or not his theory agrees with experiment. By inspection of the resulting fringe pattern, we can tell if just one mode participates in the spanwise direction or not. Similarly, we can observe readily if the analytical solution has converged in the streamwise direction. If the flutter analysis involves the finite-element approach,[11] a direct comparison between theory and experiment still can be made, simply by calculating the velocity distribution and comparing it with that measured holographically.

If the flutter is nonlinear, or nonsteady in time, we still can record the instantaneous velocity distribution by a single differential hologram. For nonsteady phenomena, the only useable information is an instanteous measurement anyway.

Time Delay, Δ

For flutter with a frequency ω it was stated earlier that the time delay must be such that

$$\omega\Delta << 1$$

for the analysis to hold. If the flutter frequency is 100 cps, we have the requirement that

$$\Delta \ll 1/2\pi(100)] \approx 1.60 \times 10^{-3} \text{sec}$$

This is not a difficult requirement to meet, since a repetitively pulsed laser can be made to emit pulses which are separated in time by as little as 1-2 μsec (10^{-6} sec). The experiments reported in Ref. 12 used pulse spacings of 25 and 50 μsec.

A more serious restriction on Δ comes from Eq. (29) and (30). In order to neglect the terms of order Δ^2, the following inequality must be satisfied

$$\left| \tfrac{1}{2}\frac{\partial^2 w}{\partial t^2}\Delta^2 \right| << \left| \frac{\partial w}{\partial t}\Delta \right| \qquad (31)$$

Equation (31) can be put into the form of a limitation on Δ

$$\Delta << \left| \frac{2(\partial w/\partial t)}{(\partial^2 w/\partial t^2)} \right| \qquad (32)$$

Ther requirement (32) usually can be met at all points where the velocity is nonzero. However, if there are localities in the velocity distribution $\dot{w}(x,y,t_1)$ where $\dot{w}=0$, the local fringe pattern will respond to the acceleration term $\tfrac{1}{2}(\partial^2 w/\partial t^2)\Delta^2$ instead of to the velocity. In all likelihood, problems of this type can be avoided easily by choosing the time t_1 to be such that the entire panel surface has a velocity $\dot{w}$.

If all else fails, we can ask the structural analyst to compute

$$\dot{w}(x,y,t_1) + (\Delta/2)\ddot{w}(x,y,t_1) \qquad (33)$$

for comparison with the holographic results.

Concluding Remarks

The analysis demonstrates that holography offers a potential means of obtaining the deflection or velocity of a fluttering panel, regardless of the panel shape, boundaries, or flutter mode. A single differential hologram will provide the experimental data required to verify (or disprove) a flutter analysis as regards the spatial distribution of the flutter mode. Although the present study has been focused on panel flutter, similar pulsed holographic techniques might be applied to the general flutter problem e.g., flutter of an aeroelastic aircraft model, transonic flutter of aerodynamic surfaces, etc. It is hoped that the results outlined here will inspire aeroelasticians to investigate and use holography as a measurement tool.

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